

K or L

Dandy

THE WEST AFRICAN EXAMINATIONS COUNCIL
WASSCE FOR SCHOOL CANDIDATES, 2023
FINAL MARKING SCHEME

FURTHER MATHEMATICS / MATHEMATICS (ELECTIVE) 2 (ESSAY)

GENERAL INSTRUCTIONS

1. Marks are subdivided into method (**M**) marks, accuracy (**A**) and independent accuracy marks not preceded by **M** mark (**B**).
2. The **M** marks should be given for a particular stage if the method is correct; that is, it would, if correctly carried out without numerical error, yield the right answer. **M** marks are not subdivided and unless the **M** marks for a stage have been awarded, no **A** marks can be gained for that stage.
3. Give **0** for results obtained for work that is indecipherable or wholly suppressed (W.S.O).
4. Deduct **1** mark for omission of units or for wrong units but this should not be more than once in the whole paper.
5. **12** questions must be answered in all. **All** the questions in Section A and **4** Questions in Section B with **at least 1** Question from each part.
6. If a question is attempted more than once, **mark all** and record the marks for that which has gained the **highest marks**.
7. If more questions are attempted than the rubric allows, delete the marks for those extra questions which have gained the **lowest marks** and ignore the remaining questions in that part by writing '**MQA**' through. ('**MQA**' denotes 'More Question Answered' than the rubric allows).
8. Unless otherwise stated, equivalent methods not specified in the Marking Scheme should be accepted and given appropriate marks.
9. The **final total marks** according to the Marking Scheme is **100**.
10. The final total (and not the question total) should be rounded upwards to the nearest whole number.

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GHANA

Question Number	Details	Marks
1.	$4x + y + x \frac{dy}{dx} - 6y \frac{dy}{dx} = 0$ $(x - 6y) \frac{dy}{dx} = -4x - y$ $\frac{dy}{dx} = \frac{-4x - y}{x - 6y}$ $\frac{dy}{dx}(1,6) = \frac{-4(1) - 6}{1 - 6(6)}$ $= \frac{-10}{-35}$ $\frac{dy}{dx} = \frac{2}{7}$	M1 for differentiating (one term correct) A1 for correct differentiation (All terms correct) M1 for simplifying A1 for $\frac{dy}{dx} = \frac{-4x - y}{x - 6y}$ or equivalent M1 for substituting A1 for $\frac{2}{7}$ [6 Marks]
2.	$a = 54, \quad S_{\infty} = 162$ $162 = \frac{54}{1-r}$ $54 = 162 - 162r$ $162r = 108$ $r = \frac{108}{162}$ $r = \frac{2}{3}$ $U_5 = ar^4 = 54 \left(\frac{2}{3}\right)^4$ $U_5 = \frac{32}{3}$ $U_6 = ar^5 = 54 \left(\frac{2}{3}\right)^5$ $U_6 = \frac{64}{9}$ $U_5 + U_6 = \frac{32}{3} + \frac{64}{9}$ $= \frac{160}{9}$ $\therefore U_5 + U_6 = 17.778 \text{ or } 17\frac{7}{9}$	M1 for substituting A1 for $r = \frac{2}{3}$ B1 for $\frac{32}{3}$ B1 for $\frac{64}{9}$ M1 for adding A1 for 17.778 or $17\frac{7}{9}$ [6 Marks]

NB: Don't accept improper fractions

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GHANA

Question Number	Details	Marks
3.	$\sqrt{q-3} + \sqrt{q+2} = 5$ $q-3 + q+2 + 2\sqrt{(q-3)(q+2)} = 25$ $2q-1 + 2\sqrt{(q-3)(q+2)} = 25$ $2\sqrt{(q-3)(q+2)} = 26-2q$ $\sqrt{(q-3)(q+2)} = 13-q$ $(q-3)(q+2) = (13-q)^2$ $q^2 - q - 6 = 169 - 26q + q^2$ $-q + 26q = 169 + 6$ $25q = 175$ $q = 7$	M1 for squaring A1 for correct squaring M1 for squaring A1 for correct squaring M1 for solving for q A1 for $q = 7$ [6 Marks]
4.(a)	$9\sin 2\theta - 28\cos 2\theta = 0$ $9\sin 2\theta = 28\cos 2\theta$ $\frac{\sin 2\theta}{\cos 2\theta} = \frac{28}{9}$ $\tan 2\theta = \frac{28}{9}$ $2\theta = \tan^{-1}\left(\frac{28}{9}\right)$ $2\theta = 72.1811^\circ$ $\theta = 36.0906^\circ$ $\theta = 36.09^\circ$	M1 for dividing M1 for finding θ A1 for $\theta = 36.09^\circ$
(b)	$(3r - 2s)^4 = (3r)^4 + \binom{4}{3}(3r)^3(-2s) + \binom{4}{2}(3r)^2(-2s)^2$ $+ \binom{4}{1}(3r)^1(-2s)^3 + (-2s)^4$ $= 81r^4 - 216r^3s + 216r^2s^2 - 96rs^3 + 16s^4$	M1 for expanding A2 for answer $(-\frac{1}{2}ee)$ [6 Marks] <i>At least 2 terms correct</i>

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GHANA

CANDIDATE'S NAME

Question Number	Details	Marks
5.(a)	$P(\text{speak Twi}) = \frac{3}{20}$ $P(\text{does not speak Twi}) = \frac{17}{20}$ $P(\text{more than half speak Twi}) = P(X > 2)$ $= \binom{4}{3} (0.15)^3 (0.85)^1 + \binom{4}{4} (0.15)^4 (0.85)^0$ $= 0.011475 + 0.00050625$ $= 0.01198125$ $= 0.012 \text{ or } \frac{1917}{160000}$	B1 for $\frac{17}{20}$ or 0.85 or 85% M1 for forming probability [at least one term correct] M1 for simplifying A1 for 0.012 or $\frac{1917}{160000}$
(b)	$P(\text{at most speak Twi}) = P(X \leq 2)$ $= \binom{4}{0} (0.15)^0 (0.85)^4 + \binom{4}{1} (0.15)^1 (0.85)^3$ $+ \binom{4}{2} (0.15)^2 (0.85)^2$ $= 0.52200625 + 0.368475 + 0.0975375$ $= 0.98801875$ $= 0.988 \text{ or } \frac{158083}{160000}$	M1 for forming probability [one term correct] A1 for 0.988 or $\frac{158083}{160000}$

[6 Marks]

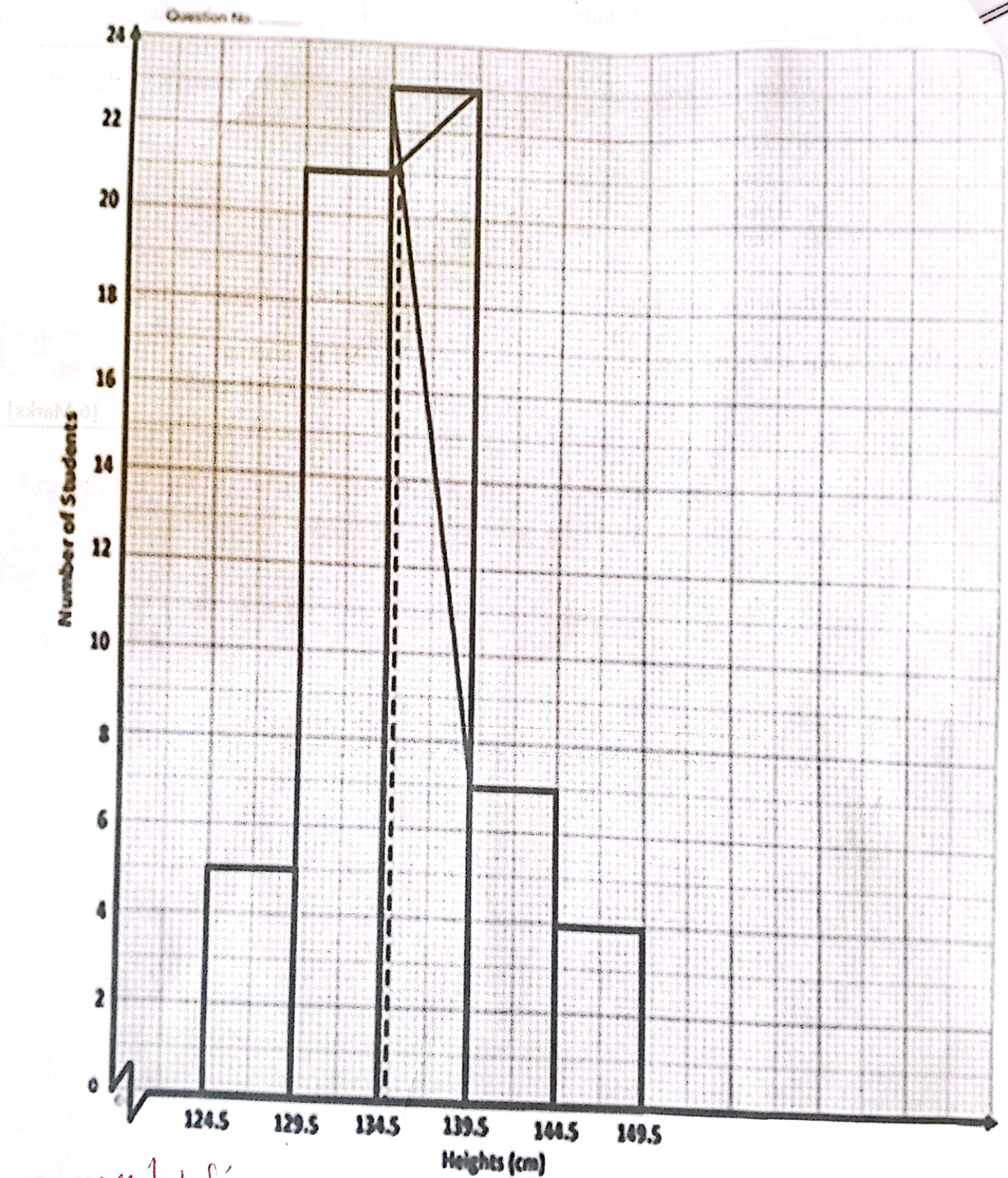
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FURTHER MATHEMATICS / MATHEMATICS (ELECTIVE) 2 (ESSAY)
GHANA

Question Number	Details	Marks																		
6. (a)	<table border="1"> <thead> <tr> <th>Height</th><th>Frequency</th><th>Class Boundary</th></tr> </thead> <tbody> <tr> <td>125 – 129</td><td>5</td><td>124.5 – 129.5</td></tr> <tr> <td>130 – 134</td><td>21</td><td>129.5 – 134.5</td></tr> <tr> <td>135 – 139</td><td>23</td><td>134.5 – 139.5</td></tr> <tr> <td>140 – 144</td><td>7</td><td>139.5 – 144.5</td></tr> <tr> <td>145 – 149</td><td>4</td><td>144.5 – 149.5</td></tr> </tbody> </table>	Height	Frequency	Class Boundary	125 – 129	5	124.5 – 129.5	130 – 134	21	129.5 – 134.5	135 – 139	23	134.5 – 139.5	140 – 144	7	139.5 – 144.5	145 – 149	4	144.5 – 149.5	No marks for the table
Height	Frequency	Class Boundary																		
125 – 129	5	124.5 – 129.5																		
130 – 134	21	129.5 – 134.5																		
135 – 139	23	134.5 – 139.5																		
140 – 144	7	139.5 – 144.5																		
145 – 149	4	144.5 – 149.5																		
(b)	See the histogram attached Mode = 135.0 ± 0.5	B4 for histogram (-1ee) M1 for reading A1 for 135.0 ± 0.5 [Evidence from graph] [6 Marks]																		

Note: Where question doesn't specify number of decimal places, accept AT LEAST ONE decimal place

- If graph's origin is not from zero, deduct 1

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- Also wrong labeling of axes = -1
- Wrong scaling = -1
- IF no table, still mark the graph
- Use only class midpoint w boundaries. IF not, zero [BO]
- IF labeling doesn't start from 0, give BO

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GHANA

Question Number	Details	Marks
7.	$\text{Resultant force} = (2i + j) + (-3i + 4j) + (4i - 6j)$ $= 3i - j$ $3i - j = 12a$ $a = \frac{1}{4}i - \frac{1}{12}j$ $ a = \sqrt{\left(\frac{1}{4}\right)^2 + \left(-\frac{1}{12}\right)^2}$ $= 0.263523138$ $ a = 0.2635$	M1 for adding <i>[give total marks for just adding]</i> A1 for $3i - j$ M1 for finding acceleration <i>because</i> A1 for $a = \frac{1}{4}i - \frac{1}{12}j$ <i>there's mistake in question</i> M1 for finding magnitude A1 for 0.2635
		[6 Marks]
8.(a)	$a = \left(\frac{3}{2} - \frac{1}{10}t\right) \text{ms}^{-2}$ $v = \int \left(\frac{3}{2} - \frac{1}{10}t\right) dt$ $v = \frac{3}{2}t - \frac{1}{20}t^2$ After 10 secs, $v = \frac{3}{2}(10) - \frac{1}{20}(10)^2$ $= \frac{30}{2} - \frac{100}{20}$ $v = 10\text{ms}^{-1}$	M1 for integrating <i>[one term correct]</i> A1 for $\frac{3}{2}t - \frac{1}{20}t^2$ M1 for substituting A1 for 10ms^{-1}
(b)	$v = \frac{3}{2}t - \frac{1}{20}t^2$ $S = \int \left(\frac{3}{2}t - \frac{1}{20}t^2\right) dt$ $S = \frac{3}{4}t^2 - \frac{1}{60}t^3$	M1 for integrating <i>[one term correct]</i> A1 for $S = \frac{3}{4}t^2 - \frac{1}{60}t^3$
		[6 Marks]

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GHANA

Question Number	Details	Marks
9.(a)	Four French teachers Number of ways = $\binom{6}{4} \times \binom{6}{1}$ $= 15 \times 6$ $= 90 \text{ ways}$	M1 for forming combination M1 for simplifying A1 for 90
(b)	At least one French teacher Number of ways = $\binom{6}{1} \times \binom{6}{4} + \binom{6}{2} \times \binom{6}{3}$ $+ \binom{6}{3} \times \binom{6}{2} + \binom{6}{4} \times \binom{6}{1} + \binom{6}{5} \times \binom{6}{0}$ $= (6 \times 15) + (15 \times 20) + (20 \times 15)$ $+ (15 \times 6) + (6 \times 1)$ $= 90 + 300 + 300 + 90 + 6$ $= 786 \text{ ways}$	M1 for forming combination <i>[one term correct]</i> A2 for correct combination(-1ee) M1 for simplifying A1 for 786
(c)	At most two Music teachers Number of ways = $\binom{6}{3} \times \binom{6}{2} + \binom{6}{4} \times \binom{6}{1}$ $+ \binom{6}{5} \times \binom{6}{0}$ $= (20 \times 15) + (15 \times 6) + (6 \times 1)$ $= 300 + 90 + 6$ $= 396 \text{ ways}$	M1 for forming combination <i>[one term correct]</i> A2 for correct combination(-1ee) M1 for simplifying <i>[one term correct]</i> A1 for 396

[13 Marks]

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Question Number	Details	Marks
10.(a)	Hemisphere, $v = \frac{2}{3}\pi r^3$ $\frac{dv}{dr} = 2\pi r^2$ $\frac{dv}{dt} = 2\pi cm^3 s^{-1}$ $r = 5 \text{ cm}$ $21\pi cm^3 s^{-1} = 2\pi r^2 \times \frac{dr}{dt}$ $21\pi cm^3 s^{-1} = 2\pi \times 5^2 \times \frac{dr}{dt}$ $21\pi cm^3 s^{-1} = 50\pi cm^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{21\pi cm^3 s^{-1}}{50\pi cm^2}$ $\frac{dr}{dt} = \frac{21}{50} cms^{-1} \text{ or } 0.42 cms^{-1}$ $3v - w = 4$	$\frac{1}{2} \left[\frac{4}{3} \pi r^3 \right] \frac{1}{2} \text{ of sphere}$ M1 for differentiating A1 for $2\pi r^2$ M1 for substituting M1 for simplifying A1 for $\frac{dr}{dt} = \frac{21}{50} cms^{-1} \text{ or } 0.42 cms^{-1}$
(b)	$3v - 4 = w \dots (1)$ $v^2 + 2vw - w^2 = -2 \dots (2)$ Substitute for w in equation (2) $v^2 + 2v(3v - 4) - (3v - 4)^2 = -2$ $v^2 + 6v^2 - 8v - (9v^2 - 24v + 16) = -2$ $7v^2 - 8v - 9v^2 + 24v - 16 = -2$ $-2v^2 + 16v - 14 = 0$ $v^2 - 8v + 7 = 0$ $v(v - 7) - 1(v - 7) = 0$ $(v - 7)(v - 1) = 0$ $\therefore v = 7 \text{ or } v = 1$ When $v = 7, w = 3(7) - 4$ $= 17$ When $v = 1, w = 3(1) - 4$ $= -1$	M1 for substituting M1 for simplifying A1 for $v^2 - 8v + 7 = 0$ M1 for finding v A1 for $v = 7 \text{ or } v = 1$ M1 for substituting (either 7 or 1) A1 for $w = 17$ A1 for $w = -1$

[13 Marks]

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rest is given by a

Question Number	Details	Marks
11.(a)	$3x^2 + 4x - 5 = 0$ $\alpha + \beta = -\frac{4}{3}$ $\alpha\beta = -\frac{5}{3}$ $\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$ $= -\frac{4}{3}\left[\left(\frac{4}{3}\right)^2 - 3\left(-\frac{5}{3}\right)\right]$ $= -\frac{4}{3}\left[\frac{16}{9} + \frac{15}{3}\right]$ $= -\frac{4}{3}\left[\frac{61}{9}\right]$ $= -\frac{244}{27}$ $\alpha^3\beta^3 = \left(-\frac{5}{3}\right)^3$ $= -\frac{125}{27}$ $x^2 + \frac{244}{27}x - \frac{125}{27} = 0$ $27x^2 + 244x - 125 = 0$	 B1 for $-\frac{4}{3}$ B1 for $-\frac{5}{3}$ M1 for substituting M1 for simplifying A1 for $-\frac{244}{27}$ M1 for substituting A1 for $-\frac{125}{27}$ M1 for substituting A1 for $27x^2 + 244x - 125 = 0$
(b)	$5^x \times 5^{x+1} = 5^{3-x} \div (125^{-1} \times 5^{-x-1})$ $5^{2x} \times 5^1 = \frac{5^{3-x}}{5^{-3} \times 5^{-x-1}}$ $5^{2x+1} = \frac{5^{3-x}}{5^{-4} \times 5^{-x}}$ $5^{2x+1} = 5^{3+4-x+x}$ $5^{2x+1} = 5^7$ $2x + 1 = 7$ $2x = 6$ $x = 3$	 M1 for simplifying A1 for correct simplification M1 for solving A1 for $x = 3$

[13 Marks]

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Question Number	Details	Marks
12.(a)	$p = \frac{2}{3}, q = \frac{1}{3}, n = 6$ $P(\text{half survived}) = P(X = 3)$ $= \binom{6}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^3$ $= \frac{160}{729} \text{ or } 0.2195$	B1 for $\frac{2}{3}$ M1 for forming probability M1 for simplifying A1 for $\frac{160}{729}$ or 0.2195
(b)	$P(\text{more than two - third survived}) = P(X > 4)$ $= \binom{6}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + \binom{6}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0$ $= \frac{64}{243} + \frac{64}{729}$ $= \frac{256}{729} \text{ or } 0.3512$	M1 for forming probability <i>[one term correct]</i> A2 for correct probability <i>(-1 ee)</i> M1 for simplifying A1 for $\frac{256}{729}$ or 0.3512 <i>NB: 0.3511 is wrong. Let decimal place be correct when</i> If decimal place with Speeches 0.4 ✓ 0.35 ✓ 0.351 ✓ 0.3511 X
(c)	$P(\text{less than one - third died}) = P(X < 2)$ $= \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6$ $= \frac{64}{243} + \frac{64}{729}$ $= \frac{256}{729} \text{ or } 0.3512$	M1 for forming probability <i>[one term correct]</i> A1 for correct probability <i>(-1/2 ee)</i> M1 for simplifying A1 for $\frac{256}{729}$ or 0.3512

[13 Marks]

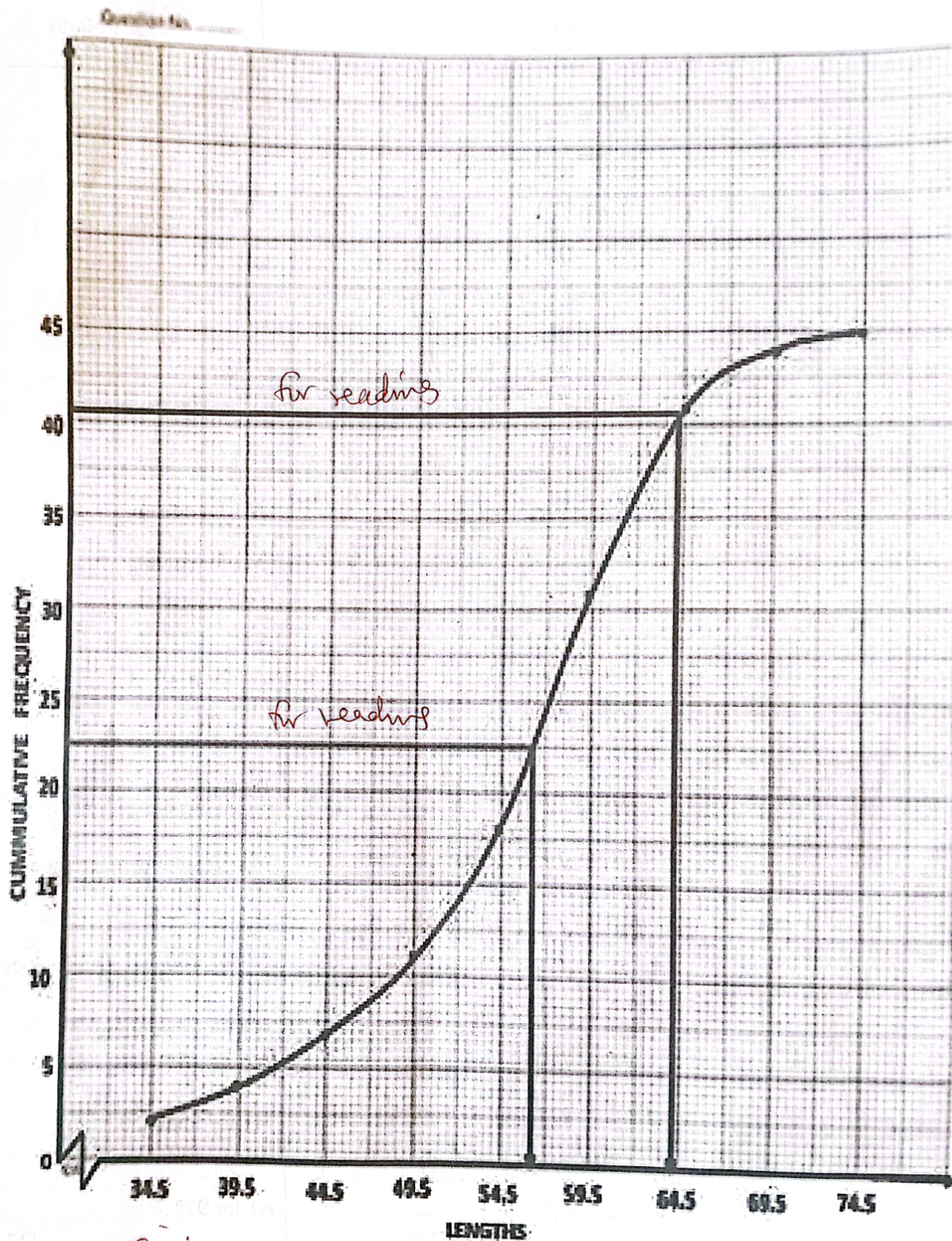
- The probability, if terms are more than one, cancel out mult simply without jumping.
- However, if just a term, its accepted to 'jump'

GHANA

Question Number	Details	Marks																				
13.(a)	$2 + 2 + q - 1 + 4 + 3q - 5 + q^2 - 3 + 14 + 3 + 1 = 45$ $q^2 + 4q + 13 = 45$ $q^2 + 4q - 32 = 0$ $q^2 + 8q - 4q - 32 = 0$ $(q + 8)(q - 4) = 0$ $q = 4 \text{ or } -8$ $q = 4$ <table border="1"> <thead> <tr> <th>Length</th><th>Frequency (f)</th></tr> </thead> <tbody> <tr><td>30 - 34</td><td>2</td></tr> <tr><td>35 - 39</td><td>2</td></tr> <tr><td>40 - 44</td><td>3</td></tr> <tr><td>45 - 49</td><td>4</td></tr> <tr><td>50 - 54</td><td>7</td></tr> <tr><td>55 - 59</td><td>13</td></tr> <tr><td>60 - 64</td><td>10</td></tr> <tr><td>65 - 69</td><td>3</td></tr> <tr><td>70 - 74</td><td>1</td></tr> </tbody> </table>	Length	Frequency (f)	30 - 34	2	35 - 39	2	40 - 44	3	45 - 49	4	50 - 54	7	55 - 59	13	60 - 64	10	65 - 69	3	70 - 74	1	M1 for equating A1 for $q^2 + 4q - 32 = 0$ M1 for solving A1 for $q = 4$ <i>If candidate uses calculator to solve the quadratic, give zero</i> B1 for frequency column ($-\frac{1}{2}ee$)
Length	Frequency (f)																					
30 - 34	2																					
35 - 39	2																					
40 - 44	3																					
45 - 49	4																					
50 - 54	7																					
55 - 59	13																					
60 - 64	10																					
65 - 69	3																					
70 - 74	1																					
(b)	<table border="1"> <thead> <tr> <th>Less than</th><th>Cumulative Frequency</th></tr> </thead> <tbody> <tr><td>34.5</td><td>2</td></tr> <tr><td>39.5</td><td>4</td></tr> <tr><td>44.5</td><td>7</td></tr> <tr><td>49.5</td><td>11</td></tr> <tr><td>54.5</td><td>18</td></tr> <tr><td>59.5</td><td>31</td></tr> <tr><td>64.5</td><td>41</td></tr> <tr><td>69.5</td><td>44</td></tr> <tr><td>74.5</td><td>45</td></tr> </tbody> </table> See the cumulative frequency curve attached	Less than	Cumulative Frequency	34.5	2	39.5	4	44.5	7	49.5	11	54.5	18	59.5	31	64.5	41	69.5	44	74.5	45	B2 for cumulative frequency column ($-\frac{1}{2}ee$)
Less than	Cumulative Frequency																					
34.5	2																					
39.5	4																					
44.5	7																					
49.5	11																					
54.5	18																					
59.5	31																					
64.5	41																					
69.5	44																					
74.5	45																					
(c) (i)	Median length = $56.0 \pm 0.5 \text{ cm}$	B2 for graph ($-\frac{1}{2}ee$) M1 for reading median [graph]																				
(ii)	90 th percentile = $64.0 \pm 0.5 \text{ cm}$	A1 for $56.0 \pm 0.5 \text{ cm}$ M1 for reading 90th percentile [graph] A1 for $64.0 \pm 0.5 \text{ cm}$																				
		[13 Marks]																				

ram in 6(a) to estimate
mark is

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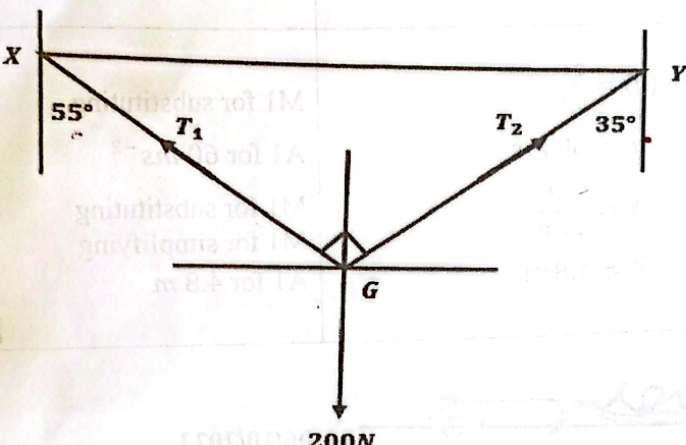


- Origin shd be zero else: -1
- All other errors on graph: $-\frac{1}{2}$
- Labeling = $-\frac{1}{2}$
- wrong scaling: $-\frac{1}{2}$

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Question Number	Details	Marks
14. (a)	$\overrightarrow{KL} = \overrightarrow{OL} - \overrightarrow{OK}$ $= \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\overrightarrow{KL} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR}$ $= \begin{pmatrix} 6 \\ -3 \end{pmatrix} - \begin{pmatrix} m-1 \\ n-2 \end{pmatrix}$ $= \begin{pmatrix} 6-(m-1) \\ -3-n+2 \end{pmatrix}$ $\overrightarrow{RQ} = \begin{pmatrix} 7-m \\ -1-n \end{pmatrix}$ $\overrightarrow{KL} = \overrightarrow{RQ}$ $\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 7-m \\ -1-n \end{pmatrix}$ $3 = 7 - m$ $m = 4$ $2 = -1 - n$ $n = -3$	B1 for $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ B1 for $\begin{pmatrix} 7-m \\ -1-n \end{pmatrix}$ M1 for equating M1 for solving A1 for $m = 4$ A1 for $n = -3$
(b)	$\overrightarrow{KR} = \overrightarrow{OR} - \overrightarrow{OK}$ $= \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\overrightarrow{KR} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$ $ \overrightarrow{KR} = \sqrt{4^2 + (-8)^2}$ $ \overrightarrow{KR} = \sqrt{80}$ $ \overrightarrow{KR} = \sqrt{3^2 + 2^2}$ $ \overrightarrow{KR} = \sqrt{13}$ $\begin{pmatrix} 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -8 \end{pmatrix} = (\sqrt{13})(\sqrt{80})\cos\theta$ $-4 = \sqrt{1040}\cos\theta$ $\cos\theta = \frac{-4}{32.24903}$ $\theta = \cos^{-1}\left(\frac{-4}{32.24903}\right)$ $\theta = 97^\circ$	B1 for $\begin{pmatrix} 4 \\ -8 \end{pmatrix}$ M1 for finding magnitude A1 for $\sqrt{80}$ and $\sqrt{13}$ ($-\frac{1}{2}$ ee) M1 for substituting B1 for -4 (from the dot product) M1 for finding θ A1 for 97°
		[13 Marks]

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GHANA

Question Number	Details	Marks
15.(a)	 200N $\frac{200}{\sin 90^\circ} = \frac{T_1}{\sin 145^\circ}$ $T_1 = \frac{200 \times \sin 145^\circ}{\sin 90^\circ}$ $T_1 = 114.7 \text{ N}$ $\frac{T_2}{\sin 125^\circ} = \frac{200}{\sin 90^\circ}$ $T_2 = \frac{200 \times \sin 125^\circ}{\sin 90^\circ}$ $T_2 = 163.8 \text{ N}$ ALITER $\begin{pmatrix} -T_1 \cos 35^\circ \\ T_1 \sin 35^\circ \end{pmatrix} + \begin{pmatrix} T_2 \cos 55^\circ \\ T_2 \sin 55^\circ \end{pmatrix} + \begin{pmatrix} 200 \cos 90^\circ \\ -200 \sin 90^\circ \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $-0.8192T_1 + 0.5736T_2 = 0 \quad \dots (1)$ $0.5736T_1 + 0.8192T_2 = 200 \quad \dots (2)$ $-0.4699T_1 + 0.3290T_2 = 0 \quad \dots (3)$ $0.4699T_1 + 0.6711T_2 = 163.84 \quad \dots (4)$ $T_2 = \frac{163.84}{1.0001}$ $\therefore T_2 = 163.8 \text{ N}$ $\therefore T_1 = 114.7 \text{ N}$	position of X, Y [1/2e] arrows [1/2e] 55°, 35°, [1/2] G [B0 for wrong position] Diagram is correct without G [1/2] B2 for diagram (-1/2 ee) If B0, all other calculations are 0. M1 for substituting M1 for simplifying A1 for 114.7 N M1 for substituting M1 for simplifying A1 for 163.8 N M1 for resolving [at least one term correct] A2 for correct resolution (-1 ee) M1 for solving A1 for $T_2 = 163.8 \text{ N}$ A1 for $T_1 = 114.7 \text{ N}$

WASSCE FOR SCHOOL CANDIDATES, 2023
FURTHER MATHEMATICS / MATHEMATICS (ELECTIVE) 2 (ESSAY)
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Question Number	Details	Marks
15.(b)	$\text{Deceleration } (a) = \frac{24}{0.4}$ $a = 60 \text{ ms}^{-2}$ $S = \frac{24^2}{2 \times 60}$ $S = 4.8 \text{ m}$	M1 for substituting A1 for 60 ms^{-2} M1 for substituting M1 for simplifying A1 for 4.8 m [13 Marks]

GHANA



06/10/2023

NIGERIA



06/10/2023